SUBGROUP STRUCTURE OF CHEVALLEY GROUPS OVER RINGS

ALEXEI STEPANOV

Let G be a Chevalley–Demazure group scheme and A a commutative ring. Let D be a subgroup of $G(A)$. We study the lattice $\mathcal{L} = L\big(D, G(A)\big)$ of subgroups of $G(A)$, containing D.

For certain types of D one can prove the *sandwich classification theorem* (SSC). We say that $\mathcal{L}$ satisfies SSC iff given $H \in \mathcal{L}$ there exists a subgroup $F \in L$ such that $D^F = F$ and H lies between F and its normaliser. In other words, $\mathcal{L}$ splits into a disjoint union of "sandwiches" $L(F, N)$, where $D^F = F$ and N is the normaliser of F.

We discuss the following items.

1. A list of types of D for which SSC is proved or is expected to hold and relations between this list and Aschbacher classes.
2. Steps of a standard proof of SSC in linear groups.
3. What is known for each type of D listed above.
4. Generalisation of the above for the lattice of subgroups, normalised by D.

Special attention will be payed for the case $D = E(S)$, where E denotes the elementary group subfunctor of G and S a subring of A. In this case I have recent results and a conjecture about the final answer.

Instead of definitions of required notions from Chevalley group theory I shall explain what they mean in SL_n and/or Sp_{2n}. Specific knowledge about Chevalley groups (even the definition of a Chevalley–Demazure group scheme) is not required.