STRUCTURE OF CHEVALLEY GROUPS OVER RINGS

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Let G be a Chevalley–Demazure group scheme and R a commutative ring (one can replace G by SL_n; the techniques and results are still nontrivial). Let E be the elementary subgroup of G, i. e. $E(R)$ is the group generated by all root unipotent elements (if $G = SL_n$, then $E(R)$ is the set of all matrices that can be reduced to identity by elementary transformations).

We study the following problems:

(1) Normality of the elementary subgroup;
(2) Standard commutator formulas;
(3) Multiple relative commutator formula;
(4) Nilpotent structure of $K_1^G(R)$;
(5) Bounded word length in $E(R)$;
(6) Normal subgroups of $G(R)$.

Over a field or a local ring all the problems are easy. We use a localization procedure to reduce the problems to local rings. During the proof we state Dilation Principle (clearing denominators) and Splitting Principle. The key construction of the proof is the universal ring for a principal congruence subgroup corresponding to a principal ideal.