

LOCALIZATION TECHNIQUES FOR CHEVALLEY GROUPS OVER RINGS

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Let G be a Chevalley–Demazure group scheme over a ring K (one can replace G by SL_n over \mathbb{Z} ; the techniques and results are still nontrivial). Let H be a functorially defined subgroup of G , let R be a ring and $a \in G(R)$. A philosophy of localization is that sometimes we can deduce the fact that $a \in H(R)$ from the fact that the images of a under certain localization homomorphisms belong to H over a localized ring.

We study the following problems:

- (1) Suslin’s theorem;
- (2) Normality of the elementary subgroup;
- (3) Standard commutator formulas;
- (4) Multiple relative commutator formulas;
- (5) Normal structure;
- (6) Nilpotent structure of K_1G ;
- (7) Word length of commutators.

In view of these problems we formulate different versions of the localization method:

- (1) Original Quillen–Suslin’s method;
- (2) Adding independent variables (Taddei–Vaserstein);
- (3) Localization-completion (Bak);
- (4) Universal localization alias localization in the affine algebra of G (Stepanov);

We also consider some other tricks to be used together with or inside a localization procedure:

- (1) Splitting arguments;
- (2) Struggle against zero divisors.
- (3) Universal ring for a principle congruence subgroup corresponding to a principle ideal;
- (4) Moving a ring element inside a commutator.