SUBGROUPS OF A CHEVALLEY GROUP, CONTANING THE ELEMENTARY SUBGROUP OVER A SUBRING

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All rings are assumed to be commutative with 1. Let K be a ring, let $S \subseteq A$ be K-algebras and G an algebraic group scheme over K. This is a well known problem: to describe lattice of subgroups between $G(S)$ and $G(A)$.

The talk is about this problem for a Chevalley–Demazure group scheme $G = \mathrm{G}(\Phi, \_)$ with a root system $\Phi \neq A_1$ over $K = \mathbb{Z}$. For a ring R let $E(R) = \mathrm{E}(\Phi, R)$ denote the elementary subgroup of $G(R)$. We consider a slightly bigger lattice, namely, the lattice of subgroups between $E(S)$ and $G(A)$.

The standard description of this lattice is called standard sandwich calssification (SSC).

Definition. Fix a triple (Φ, S, A). The SSC holds if given a subgroup H between $E(S)$ and $G(A)$ there exists a unique subring R between S and A such that

$$E(R) \leqslant H \leqslant N_A(R)$$

where $N_A(R)$ denotes the normalizer of $E(R)$ in $G(A)$.

Recently I have proved that for doubly laced root systems (i. e. $\Phi = B_l, C_l, F_4$) the SSC holds for an arbitrary pair of rings provided that 2 is invertible in S. Together with another my result and a result of Ya. Nuzhin this gives a final answer to the question for which field extensions A/S and root systems the SSC holds. By simple group theoretical arguments the SSC can be extended to subgroups of $G(A)$ nomalized by $E(S)$.

I shall exhibit known results and my new results mentioned above and show the main steps of the proof illustrating them with examples of $G = \mathrm{SL}_n$ (if the step gets through for this group scheme) or $G = \mathrm{Sp}_{2n}$. Also I shall state a conjecture about the final answer and show some immediate problems in frames of this conjecture.