# Triangulation of a commutative set

**Problem.** Let $X$ be a set of pairwise commuting operators on a finite dimensional vector space $V$ over an algebraically closed field. Prove that there exists a basis of $V$ such that all elements of $X$ has upper triangular matrices in this basis.

**Gidelines.**

**Lemma 1.** *Let $A$ and $B$ be matrices such that $AB = BA$. An eigenspace of $A$ is invariant under $B$.*

**Corollary 2.** *Let $A$ and $B$ be matrices such that $AB = BA$. Then they have a common eigenvector.*

Using induction prove that that a finite commutative set of operators has a common eigenvector $u$. Extend this fact to an infinite set using that the set of all operators on $V$ is a finite dimensional vector space.

Pass to the quotient space $V/\langle u \rangle$ and apply the previous proposition to operators on this space. Prove the desired result by induction on $\dim V$.