Var. 1 (1)

1. Given columns $a = (2, 4, -2, -1)^T$ and $b = (-1, -2, 5, 5)^T$ present b as a sum of two orthogonal columns one of them being parallel to a.
2. Compute the coordinates of $x = (2, 2, 3)^T$ in *orthogonal* basis $f_1 = (-2, 2, -1)^T$, $f_2 = (-1, -2, -2)^T$, $f_3 = (2, 1, -2)^T$
3. For each of the given polinomials determine can it be a characteristic polynomial of a symmetric matrix.
a) $x^2 - 3x + 1$; b) $x^2 + 10x + 25$; c) $x^2 + 3$; d) $x^2 - 4x + 6$; e) $x^2 - 8x + 15$.
4. Let A be a symmetric 2 by 2 matrix with different eigenvalues. For each pair of columns determine can they form an eigenbasis for A.
a) $\begin{pmatrix} 7 \\ -4 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; b) $\begin{pmatrix} -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \end{pmatrix}$; c) $\begin{pmatrix} -1 \\ -9 \end{pmatrix}, \begin{pmatrix} -8 \\ 8 \end{pmatrix}$; d) $\begin{pmatrix} -4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Var. 2 (1)

1. Given columns $a = (-2, 4, 3, -3)^T$ and $b = (-4, 3, 2, -4)^T$ find a column $c \in \mathbb{R}^4$, orthogonal to a such that spans $\langle a, c\rangle$ and $\langle a, b\rangle$ are equal.
2. Compute the coordinates of $x = (2, -1, 5)^T$ in *orthogonal* basis $f_1 = (3, -2, 1)^T$, $f_2 = (-1, -4, -5)^T$, $f_3 = (-1, -1, 1)^T$
3. For each of the given polinomials determine can it be a characteristic polynomial of a symmetric matrix.
a) $x^2 - 5x + 8$; b) $x^2 - 5x + 2$; c) $x^2 - 6x + 5$; d) $x^2 + 7$; e) $x^2 - 2x + 1$.
4. Let A be a symmetric 2 by 2 matrix with different eigenvalues. For each pair of columns determine can they form an eigenbasis for A.
a) $\begin{pmatrix} 4 \\ -7 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}$; b) $\begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -9 \end{pmatrix}$; c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$; d) $\begin{pmatrix} -4 \\ -7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Var. 3 (1)

1. Given columns $a = (1, -2, -5, 2)^T$ and $b = (2, 3, 4, -5)^T$ present b as a sum of two orthogonal columns one of them being parallel to a.
2. Compute the coordinates of $x = (5, -1, -3)^T$ in *orthogonal* basis $f_1 = (1, 2, 3)^T$, $f_2 = (-5, 4, -1)^T$, $f_3 = (2, 2, -2)^T$
3. For each of the given polinomials determine can it be a characteristic polynomial of a symmetric matrix.
a) $x^2 - 2x - 3$; b) $x^2 - 8x - 2$; c) $x^2 + 6$; d) $x^2 - x + 3$; e) $x^2 - 10x + 25$.
4. Let A be a symmetric 2 by 2 matrix with different eigenvalues. For each pair of columns determine can they form an eigenbasis for A.
a) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 9 \end{pmatrix}$; b) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; c) $\begin{pmatrix} -3 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \end{pmatrix}$; d) $\begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -6 \end{pmatrix}$.

Var. 4 (1)

1. Given columns $a = (-3, -2, 3, -2)^T$ and $b = (-2, 1, 4, -5)^T$ find a column $c \in \mathbb{R}^4$, orthogonal to a such that spans $\langle a, c\rangle$ and $\langle a, b\rangle$ are equal.
2. Compute the coordinates of $x = (1, 4, 4)^T$ in *orthogonal* basis $f_1 = (5, 4, -1)^T$, $f_2 = (-1, 2, 3)^T$, $f_3 = (3, -3, 3)^T$
3. For each of the given polinomials determine can it be a characteristic polynomial of a symmetric matrix.
a) $x^2 + x - 3$; b) $x^2 + 2x + 5$; c) $x^2 - 5x + 6$; d) $x^2 - 8x + 16$; e) $x^2 + 3$.
4. Let A be a symmetric 2 by 2 matrix with different eigenvalues. For each pair of columns determine can they form an eigenbasis for A.
a) $\begin{pmatrix} 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ 8 \end{pmatrix}$; b) $\begin{pmatrix} -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$; c) $\begin{pmatrix} 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$; d) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \end{pmatrix}$.