A Markov chain

Problem. A point moves randomly over the vertices of n-polygon. Each step it moves to one of two neighboring vertices with the probability $1/2$. Compute the probabilities for the point to be in different vertices after m steps.

Gidelines. Enumerate the vertices of the polygon from 1 up to n. Consider $n \times n$ matrix $A(k)$ whose (i, j)-th entry equals to the probability of moving from the vertex i to the vertex j in k steps. The matrix $A = A(1)$ is given, and the matrix $A(m)$ is what you want to compute. By the rules of probability theory one proves that $A(k)A(l) = A(k + l)$, thus, $A(m) = A^m$.

The usual way to compute a power of a matrix is to find its eigenvalues and eigenvectors, put it into (since it is symmetric) diagonal form, take a power of this diagonal form, and change the basis back. The only problem you have is to find eigenvalues and eigenvectors of A. For this end you should guess the eigenvectors of A using the following lemma as a hint.

Lemma 1. *Let ε be a n-th root of 1. Then $\varepsilon^{k-1} + \varepsilon^{k+1} = \lambda\varepsilon^k$ where $\lambda \in \mathbb{R}$ does not depend on k.*