Var. 1 (1)

1. Write the matrix of a linear operator L in a basis u, if L acts on the basic vectors in the following way: $L(u_1) = 2u_1 - u_3$; $L(u_2) = u_1 + u_2 + u_3$; $L(u_3) = 2u_3$.
2. Find the matrix of the operator L in the standard basis of $\mathbb{R}^3$, where $L(v) = v \times (10, -5, 7)^T$ (here $\times$ denotes the vector product).
3. Let V be the vector space of all symmetric polynomials of degree at most 2 over $\mathbb{R}$ in two variables x and y. Choose a basis in V and find the matrix of the operator $L : V \to V$ in this basis, where L is given by $L(f)(x, y) = (-2x - y)\frac{\partial f}{\partial x} + (-x - 2y)\frac{\partial f}{\partial y}$.

Var. 2 (1)

1. Write the matrix of a linear operator L in a basis u, if L acts on the basic vectors in the following way: $L(u_1) = u_1 + u_2 + u_3$; $L(u_2) = -u_1 + 2u_2$; $L(u_3) = u_2 + u_3$.
2. Find the matrix of the operator L in the standard basis of $\mathbb{R}^3$, where $L(v) = v - 2\frac{(a,v)}{(a,a)}a$ and $a = (3, -2, -5)^T$ (here $(\ , \ )$ denotes the standard scalar product).
3. Let V be the vector space of real 2×2 matrices with zero trace. Choose a basis in V and find the matrix of the operator $L : V \to V$ in this basis, where L is given by $L(A) = \begin{pmatrix} 0 & -2 \\ -3 & 3 \end{pmatrix} \cdot A - A \cdot \begin{pmatrix} 0 & -2 \\ -3 & 3 \end{pmatrix}$.

Var. 3 (1)

1. Write the matrix of a linear operator L in a basis u, if L acts on the basic vectors in the following way: $L(u_1) = u_1 - 2u_2$; $L(u_2) = -u_1 + u_2 + u_3$; $L(u_3) = -u_1 + u_3$.
2. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the projector onto the axe OY. Find its matrix in the standard basis.
3. Let $V = \{p(t)\sin(5t) + q(t)\cos(5t) \,|\, p, q$ – are polynomials, of degree at most $1\}$ be the linear space over $\mathbb{R}$. Choose a basis in V and find the matrix of the operator $L : V \to V$ in this basis, where $L(p) = p'$.

Var. 4 (1)

1. Write the matrix of a linear operator L in a basis u, if L acts on the basic vectors in the following way: $L(u_1) = 2u_1 - 2u_2 + u_3$; $L(u_2) = -u_1 + u_2 + u_3$; $L(u_3) = u_1 + u_2 + u_3$.
2. Find the matrix of the operator L in the standard basis of $\mathbb{R}^3$, where $L(v) = (a, v)b - (b, v)a$, $a = (2, 1, -3)^T$, and $b = (-1, -5, 3)^T$.
3. Let V be the vector space of all real symmetric 2×2 matrices. Choose a basis in V and find the matrix of the operator $L : V \to V$ in this basis, where L is given by $L(A) = \begin{pmatrix} -1 & 1 \\ -3 & -1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} -1 & -3 \\ 1 & -1 \end{pmatrix}$.